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Eigen Decomposition based Blind Source Separation

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Abstract

Blind Source Separation (BSS) is termed as the extraction of source signals from mixed data without or with little knowledge about the source signals. This work discusses different flavors of covariance based algorithms for blind source separation. Furthermore, another heuristic variant of the algorithm is proposed which is based on the variance of observed mixtures. Proposed algorithm is tested on simulated mixed signals for recovery of sources. Results of proposed algorithm are compared with the state of the art techniques in terms of accuracy and execution time.

Keywords: Blind source separation, Covariance based source separation,

Introduction

Blind source separation belongs to the class of unsupervised learning algorithms. It is also termed as Blind signal separation and Blind signal decomposition. BSS finds its applications in different fields like medical imaging [1], Radar imaging [2, 3], Finger prints identification [4], communication [5] and audio source separation [6]. Initially BSS algorithms were only meant for one dimensional data or signals such as audio signals. However, latter on BSS algorithms were tailored and are now widely used on multidimensional signals such as images and tensor data. There are different approaches for Blind Source Separation Algorithms such as probabilistic and information thoracic approach, examples of which are Independent Component Analysis (ICA) [7] and Principal component Analysis (PCA)[8]. Other approaches which are based on structural constraints of the sources such as Non-negative matrix factorization [9]. These structural constraints on source signals are heuristics, however they gives good results.

The most common approach for Blind source separation is ICA in which it is assumed that mixing sources are independent statistically. Thus ICA finds sources from mixtures by maximizing the independence of sources. ICA algorithm may depend on one of the two broad classes ie maximization of non-Gaussianity and minimization of mutual information. The maximization of non-Gaussainity is based on central limit theorem ie Kurtosis and Negentropy while minimization of mutual information is based on Kull back-Leibler-Divergence and maximization of entropy. Most well known ICA algorithms are Infomax, JADE and FastICA.

PCA is the other well known approach for Blind Source Separation. PCA is basically a technique which transforms the correlated variables in to a set of uncorrelated variables which are called principal components.

Non-negative Matrix Factorization (NMF) is another method in which the observed mixed data is decomposed in to sources and mixing matrix with the constraints that sources and mixing matrix only consists of non-negative entries.

Other methods include Singular Value Decomposition (SVD) [10], Dependent Component Analysis [11] and sparse component analysis [12] which can be used for the solution of BSS problem under certain conditions.

Mathematical Model of BSS Algorithm

In this section we are going to discuss mathematical model of BSS along with different solutions. Let us consider an observation mixture U of order mxk such that

$$U = AS \qquad (1)$$

Where A is the mixing matrix of order mxn and S is the source matrix of order nxk.

Now the requirement is to find the un-mixing matrix W such that

$$\boldsymbol{\hat{S}} = [\boldsymbol{W}][\boldsymbol{U}] \qquad (2)$$

Where $\boldsymbol{W} = \boldsymbol{A^{-1}}$ with the constraint that $\boldsymbol{W^T W} = \boldsymbol{I}$.

Normally there is a pre-processing step in most of the BSS algorithms which is termed as ZMW which means zero mean and whitening of the observed data.

$$U' = U - E[U] \qquad (3)$$

Where U and U' represents the observed mixed data and centered data or zero mean data respectively.

Next pre-processing step is to make the data white. For whitening the data, Eigen decomposition is used [13] in which a whitening matrix O is multiplied with the observed data such that

$$Z = OU' \qquad (4)$$

Where O is the whitening matrix and is calculated by Eigen decomposition as under

$$O = ED^{-1/2}E^{T} \text{ and } EDE^{T} = E[U'U'^{T}]$$

This step will make the covariance of the source matrix identity.

$$E[SS^{T}] = I \qquad (5)$$

Now if cost function of BSS algorithm is based on Infomax [14] which can be written as under

$$h(\boldsymbol{S}) = E[\textstyle\sum_{i=1}^{m} \ln \mathrm{p}_s(\boldsymbol{s}_i)] + ln|\boldsymbol{W}| \qquad (6)$$

Where $h(\boldsymbol{S})$, $p_s(s_i)$ and $|\boldsymbol{W}|$ are the entropy, pdf of the source matrix and determinant of the un-mixing matrix respectively.

Now to find the hidden sources we have to find the un-mixing matrix by using the cost function of Infomax

$$\boldsymbol{W(n+1)} = \boldsymbol{W(n)} + \boldsymbol{\nabla}(h) \qquad (7)$$

where $\boldsymbol{\nabla}(h) = \frac{\partial \boldsymbol{h(S)}}{\partial \boldsymbol{W}} = \frac{\partial \boldsymbol{h(WD)}}{\partial \boldsymbol{W}}$

$$\boldsymbol{\nabla}(h) = \boldsymbol{W^{-T}} - \boldsymbol{2tanh(WZ)^{T}} \qquad (8)$$

However if cost function is based on Algorithm for Multiple Source Extraction (AMUSE) [15], then equation (9) can be used for finding unknown sources from mixed data.

In AMUSE C is the singular value while V represents the diag: of the inverse of C.

Eigen Decomposition Based BSS Technique

This technique is basically inspired from AMUSE which is indirectly based on PCA. In this technique we find the higher order cumulant of the covariance matrix of observed data matrix Z which is already being pre-processed. It should be kept in mind that without taking the higher order of the covariance matrix, the recovery of the hidden sources is not optimal.

$$C = E[ZZ^{T}]^{3} \qquad (10)$$

Now finding the Eigen vectors W of the covariance matrix C and using the Eigen vectors W as the un-mixing matrix.

$$W = Eig[C] \qquad (11)$$

$$S = WZ \qquad (12)$$

It is also of interest that un-mixing matrix W is calculated directly and not in iterative fashion as is done in most of ICA algorithms

Simulation Results

To find the effectiveness of the Eigen decomposition based BSS technique, four simulated sources are linearly mixed to develop eight mixtures. Sources and their mixtures are shown in Figure 1. As discussed earlier, observed mixtures are first pre-processed. Then hidden sources are extracted using ICA Infomax algorithm, the result of which is shown in Figure 2 (b) . In Figure 2 (c) sources Extracted by AMUSE are shown, Figure 2 (d) shows recovered sources by proposed higher order Eigen decomposition based BSS algorithm. It can be seen visually that the results of the proposed algorithm are more similar to actual sources as shown in Figure 2(a) for comparison. Table 1 shows the performance results of tested algorithms in terms of execution time and correlation of extracted sources with actual sources. It can be seen that performance of the proposed algorithm is comparatively better from execution time and correlation point of view.

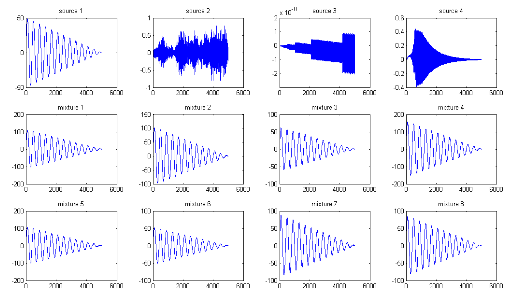

Figure 1. First row shows sources, second and third row shows mixtures

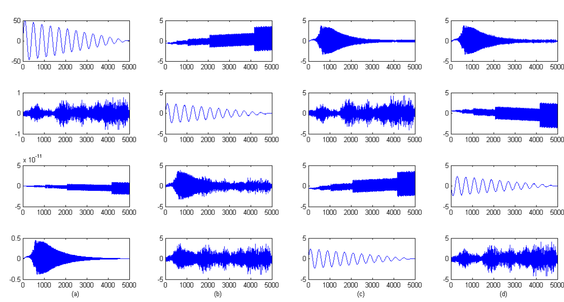

Figure 2 (a) Simulated sources (b) Sources recovered by Infomax ICA(C) Sources recovered by AMUSE (d) Sources recovered by Higher order Eigen decomposition based BSS algorithm.

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Conclusion

A mathematically simple and efficient algorithm which is based on AMUSE algorithm is presented. Performance of proposed higher order Eigen decomposition based algorithm is tested on simulated data and compared with the results of Infomax and AMUSE. Correlation and execution time results shows that proposed algorithm is relatively better than AMUSE and Infomax on this specific set of data. This algorithm can be tailored and tested on other BSS problems.

Table 1. Correlation and Execution time of proposed , Infomax and Amuse algorithms

Algorithm (Time sec)	S1	S2	S3	S4
ICA Infomax (0.8)	0.90	0.92	0.99	0.99
Amuse(0.23)	0.99	0.98	0.98	0.96
Higher order Eig: Decomp: BSS(0.20)	0.99	0.99	0.99	0.99

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